

Risk and Macro Finance Working Paper Series

No. 2014-01

Cocos, Contagion and Systemic Risk

Stephanie Chan and Sweder van Wijnbergen

University of Amsterdam and Tinbergen Institute

Risk and Macro Finance is the acclaimed research focal area of the University of Amsterdam's Faculty of Economics and Business.

The Risk and Macro Finance Working Papers Series is downloadable at <http://www.acrm.uva.nl>

Amsterdam Center of Excellence in Risk and Macro Finance
University of Amsterdam
Faculty of Economics and Business
Email: acrm@uva.nl
Webpage: <http://www.acrm.uva.nl>

Risk and Macro Finance Working Paper Series:

- [2013.1] Modelling Financial Contagion Using Mutually Exciting Jump Processes, Yacine Aït-Sahalia, Julio Cacho-Diaz and Roger J. A. Laeven, January 2013.
- [2013.2] Mutual Excitation in Eurozone Sovereign CDS, Yacine Aït-Sahalia, Roger J. A. Laeven and Loriana Pelizzon, March 2013.
- [2013.3] On Zombie Banks and Recessions after Systemic Banking Crises: It Does Matter How Governments Intervene, Timotej Homar and Sweder van Wijnbergen, April 2013.
- [2013.4] Financial Fragility, Sovereign Default Risk and the Limits to Commercial Bank Rescues, Christiaan van der Kwaak and Sweder van Wijnbergen, October 2013.
- [2014.1] Cocos, Contagion and Systemic Risk, Stephanie Chan and Sweder van Wijnbergen, August 2014.
- [2014.2] Financial Fragility and the Fiscal Multiplier, Christiaan van der Kwaak and Sweder van Wijnbergen, August 2014.

Cocos, Contagion and Systemic Risk

Stephanie Chan¹ Sweder van Wijnbergen²

August 19, 2014

Abstract

CoCo's (contingent convertible capital) are designed to convert from debt to equity when banks need it most. Using a Diamond-Dybvig model cast in a global games framework, we show that while the CoCo conversion of the issuing bank may bring the bank back into compliance with capital requirements, it will nevertheless raise the probability of the bank being run, because conversion is a negative signal to depositors about asset quality. Moreover, conversion imposes a negative externality on other banks in the system in the likely case of correlated asset returns, so bankruns elsewhere in the banking system become more probable too and systemic risk will actually go up after conversion. CoCo's thus lead to a direct conflict between micro- and macroprudential objectives. We also highlight that *ex ante* incentives to raise capital to stave off conversion depend critically on CoCo design. In many currently popular CoCo designs, wealth transfers after conversion actually flow from debt holders to equity holders, destroying the latter's incentives to provide additional capital in times of stress. Finally the link between CoCo conversion and systemic risk highlights the tradeoffs that a regulator faces in deciding to convert CoCo's, providing a possible explanation of regulatory forbearance.

JEL classification: G01, G21, G32

Keywords: Contingent Convertible Capital; Contagion; Systemic Risk; Bank Runs; Global games

¹Universiteit van Amsterdam, Tinbergen Institute. e-mail: s.chan@uva.nl

²Universiteit van Amsterdam, Tinbergen Institute. e-mail: s.j.g.vanwijnbergen@uva.nl

1 Introduction and literature review

As early as 2002, Flannery proposed an early form of contingent convertible (CoCo) capital that he called reverse convertible debentures¹. The idea was simple: whenever the bank issuing such debentures reaches a market-based capital ratio which is below a pre-specified level (say, 8% of assets), a sufficient number of said debentures would automatically convert to equity at the prevailing market price of the bank's shares. The automatic conversion feature frees the issuing bank from having to raise additional capital immediately when its capital ratio is lower than the minimum requirement. For larger shocks, conversion may not be enough to restore compliance with capital requirements, but it would make banks merely undercapitalized instead of bankrupt.

Flannery's initial CoCo design proposal was attractive as its automatic conversion feature had the potential to avoid socially costly bailouts. After the 2007 financial crisis, regulators realized that even though systemically important financial institutions (SIFIs) held Tier 2 Capital, that type of capital failed to be loss-absorbing during the time of distress. Instead, some of the SIFIs were bailed out while others were allowed to fail while in many cases for example subordinated loans continued to be serviced. In response, the Basel Committee on Banking Supervision (BCBS) made a number of changes to what is now known as the Basel 3 framework. Among the changes were the redefinition of "gone concern" to include potential bailout situations, and the inclusion of CoCo-like instruments as part of Additional Tier 1 Capital.² Also, while not yet finalized, the Basel 3 document suggested that CoCo's might play a role in ensuring that SIFIs would have higher loss absorption capacities than regular financial institutions.

The inclusion of CoCo's as part of Additional Tier 1 Capital is a likely factor in the increase of CoCo issuance. CoCo issuances totaled \$15bn in 2013, up from only \$1.7bn in 2010.³ Also, this year (2014) saw a number of banks issuing CoCo's, including Deutsche Bank and Mizuho Financial Group. Within the same period, the academic literature branched off in three different directions.

¹Unlike ordinary convertible bonds, reverse convertible debentures expose the holder to the potential downside of holding equity, as CoCo's do too.

²which must meet several requirements as set forth in the Basel 3 framework

³Financial Times: Feb 20, 2014

Flannery (2005) and McDonald (2013) were among those that dealt with design features such as triggers and bases. Pennacchi (2010) dealt with the pricing and valuation of CoCo's. Finally, Martynova and Perotti (2012) and Berg and Kaserer (2014) consider the effect of CoCo's on risk-taking incentives of banks. Moreover, several survey articles have been written about CoCo's. Maes and Schoutens (2012) provide an overview of CoCo's and enumerate the potential downside of CoCo issuance such as contagion from the banking to the insurance sector, and the creation of a "death spiral" where CoCo holders short-sell the stock of the issuing bank in order to profit from potential conversion. Avdjiev et al. (2013) discuss the features of the CoCo trend - from the reason why banks issue them to the main groups of investors that are interested in buying CoCo's, as well as the pricing of CoCo's. Wilkens and Bethke (2014) summarize and empirically assess some of the pricing models' performance. There is disagreement in the literature in particular on whether CoCo conversion should be triggered based on market prices or book values, like the various capital ratios used in the Basel-III framework. On one side are authors like Sundaresan and Wang (2014), who argue that using market prices in calculating trigger values might lead to multiple equilibria problems and potentially destabilizing bear runs on bank stock. On the other side, Calomiris and Herring (2013) argue that that problem can be mitigated by using 90-day moving averages of the particular "quasi-market data" they propose to use (market value of equity but book value of debt) while arguing that using book values actually leads to distorted incentives, for example pressure to delay recognition of losses. We do not take a position in this debate, our analysis applies to both types of triggers. Anyhow there are beyond doubt banks that have no choice because they are not listed (for example in the Netherlands 2 of the largest four banks are completely state owned (ABNAMRO and SNS Bank) while one of the remaining two has no listing either, being a cooperative (RABO)).

The effectiveness of CoCo's hinges on bank failure being caused by banks having insufficient equity to absorb losses once they have occurred. However, the majority of bank assets is funded by demand deposits. One cannot ignore the possibility that a bank may fail because depositors run before losses actually occur in anticipation of what may happen once they do occur. Jacklin and Bhattacharya (1988) and Chari and Jagannathan (1988) build on the Diamond and Dybvig

(1983) model of bank runs to show that depositors who are able to update their information about the realization of bank returns act accordingly. However, early variants of the Diamond-Dybvig model have the disadvantage that runs are zero probability events, sunspot equilibria. That makes it impossible to assess the impact of fundamentals on the probability of runs and the associated bank collapse. Goldstein and Pauzner (2005) take the Diamond-Dybvig model a substantial step further by casting the standard banking problem into a global games framework, allowing them to obtain a measure for the probability of a bank run which can be linked to fundamentals.

In this paper, we argue that a CoCo conversion conveys information that will lead depositors to update their beliefs in a manner that increases the probability of bank runs. Furthermore we examine three major types of CoCo's and show that some designs are better than others in terms of their effect on depositor run incentives. And we make a second point that is crucial for the relation between CoCo conversions and systemic risk. If different banks hold assets with correlated returns, depositors of other banks will interpret the CoCo conversion as a negative signal on their asset returns too, raising the probability of runs on banks that may not even have CoCo's and that would not have been under attack without the CoCo conversion. In other words, conversion imposes an information externality on other banks, which raises systemic risk.

This contagion channel is a second reason why we expect CoCo's to raise rather than reduce systemic risks. This is worrisome also because CoCo's are mentioned by Basel 3 as potentially useful for increasing the loss absorption capacity of SIFIs. While it is true that conversion may keep the issuing banks afloat in times of distress by immediately reducing their outstanding liability, conversion also heightens the risk that the converting banks, and other banks to the extent that they have correlated assets, will face a run.

While CoCo's have different trigger points and conversion mechanisms, many of them have a "point of nonviability" clause which effectively gives regulators control over when CoCo's convert. But regulators may end up having to make difficult choices in such circumstances. If conversion actually raises systemic risk, microprudential and macroprudential considerations may well be at variance, possibly leading to high pressure for regulatory forbearance.

Finally, while it is sometimes argued that the consequences of CoCo conversion are such that equity holders will always stave off their conversion by supplying additional capital, we show that that critically depends on CoCo design. In fact we show that in several currently popular CoCo designs, wealth transfers upon conversion actually go in the wrong direction, from junior debtors to equity holders, leading to lower rather than higher incentives to supply capital in times of distress. Only CoCo's where existing equityholders are strongly diluted by a conversion provide an incentive to supply additional capital to stave off conversion. Finally we also show that higher trigger levels do not influence direction or size of post conversion wealth transfers, but do mitigate the impact of conversion on financial fragility, although there is no trigger level at or above which the negative impact on fragility (from the regulator's point of view, i.e. conversion increases financial fragility) disappears or changes sign.

2 Basic Model

2.1 Setup

As in Diamond-Dybvig (henceforth DD), there are three periods ($t = 0, 1, 2$). There is a continuum $[0, 1]$ of agents who are each born with 1 unit of wealth at $t = 0$. The agents are risk-averse with $-cu''(c)/u'(c) > 1$. $u(0) = 0$ for all agents. A fraction λ of the agents are early consumers who can only consume at $t = 1$ with corresponding utility $u(c_1)$ while the remaining $1 - \lambda$ are late consumers who may consume at either $t = 1$ or $t = 2$, with corresponding utility $u(c_1 + c_2)$. There is no aggregate uncertainty at $t = 0$ so the proportion of early (λ) and late ($1 - \lambda$) consumers is known. However, only at $t = 1$ will the individual nature of an agent be revealed. This information is known only to the agent, so agents face idiosyncratic risk.

We depart from DD by introducing a risky technology that generates a return of $R > 1$ with probability $p(\theta)$ and 0 with probability $1 - p(\theta)$ after two periods, like in Goldstein and Pauzner (2005, GP henceforward). The investment may be liquidated at $t = 1$ without any costs other than

the foregone yield. θ is a measure of economic fundamentals such that $p(\theta)$ is strictly increasing in θ . $p(\theta) \in [0, 1]$ for any θ , where $\theta \sim U[0, 1]$.

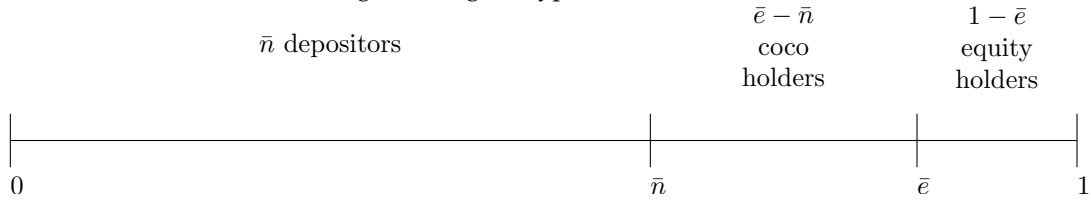
Because the risky investment can be liquidated without cost, agents are better off investing their endowment into the asset. Also, we assume that R is high enough so that $E_\theta p(\theta)u(R) > u(1)$, making it worthwhile for patient agents to wait until $t = 2$. Without any pooling of risk, the best attainable utility levels are $u(1)$ for the impatient consumers and $p(\theta)u(R)$ for the patient ones (in state of nature θ).

2.2 Bank

If there was a social planner with perfect information about agent types, he can offer higher utility levels for the agents, because idiosyncratic risk averages out upon aggregation. The social planner would offer $r_1 > 1$ to the impatient agents, and $\frac{1-\lambda r_1}{1-\lambda}R < R$ (with probability $p(\theta)$) to the patient ones. We will refer to this contract as Diamond-Dybvig contracts or DD contracts. This way, risk sharing is attained in the sense that consumers, who do not know their type yet at the moment of depositing, can transfer some income from the risky stream towards the safe stream. This is impossible in the autarkic case: because information about types is private, this solution cannot be implemented in the absence of the omniscient social planner. But DD show that a bank can mimic the socially optimal outcome by offering demand deposit contracts (which is to offer r_1 at $t = 1$ and $\frac{1-\lambda r_1}{1-\lambda}R$ with probability $p(\theta)$ at $t = 2$). Assuming r_1 is chosen well, agents will not lie about their types - only the impatient ones will withdraw at $t = 1$, so that risk sharing can be implemented. With demand deposit contracts however, the bank is subject to a sequential service constraint. It will give r_1 to those who withdraw at $t = 1$ until the asset base is exhausted. Thus, latecomers beyond the $\frac{1}{r_1}th$ withdrawing depositor will get nothing.

We depart from DD and GP by introducing two additional types of agents that also function as sources of funding for the bank: CoCo holders and equity holders. So there are only $\bar{n} < 1$ depositors. Depositors are still offered DD contracts. Let $\bar{e} - \bar{n}$ be the measure of CoCo holders, and let $1 - \bar{e}$ be the measure of equity holders such that the total measure of agents is 1. Figure 1 shows

Figure 1: Agent Types and Measures



the types of agents in the continuum.

In practice so far, issued CoCo's fall into one of three types: convert-to-equity (CE), principal writedowns (PWD), and principal writedowns coupled with a cash payout to the CoCo holders at the time the write down occurs (CASH; RABO of the Netherlands is the only bank that has issued CoCo's of this type). In the first part of our paper the distinction does not matter yet, and neither does the existence of equity because the focus is on the signal value of conversion to depositors. In the second part of the paper, we explicitly consider different types of CoCo's and the wealth transfers they imply on conversion; then equity starts playing a role too. All three types of CoCo's are non-redeemable, so CoCo holders cannot stage a run at $t = 1$. Of course, equity holders also cannot run at $t = 1$. The important thing to note is that CoCo's that do not convert are essentially long-term contracts which mature at $t = 2$, are illiquid at $t=1$ and subordinated to deposits. However if the bank survives until $t = 2$, after conversion CoCo holders may share in the gains depending on the type of CoCo issued: CE holders do and the other two types do not.) .

Even with long term funding without early withdrawal possibilities, runs are still possible as long as $\frac{1}{r_1} < \bar{n}$ which we assume to hold. We furthermore assume that the DD contracts offered by the banks are such that the incentive compatibility constraint $r_1 > p(\theta) \frac{1-\lambda r_1}{1-\lambda} R$ continues to hold: impatient consumers will always withdraw once they find out they are impatient. Finally, there is no deposit insurance in this model.

Table 1: Agent types, measures, and contributions at $t = 0$

Agent Type	Measure/Contribution
Impatient Depositors	$\lambda \bar{n}$
Patient Depositors	$(1 - \lambda) \bar{n}$
CoCo Holders	$\bar{e} - \bar{n}$
Equity Holders	$1 - \bar{e}$
Total	1

2.3 The Regulator

There is a regulator who is interested in preserving financial stability. We assume conversion occurs when the regulator decides to trigger the conversion and so forces the bank's leverage ratio down in line with the terms of the CoCo design. Note that for CoCo's to be counted as part of regulatory capital of any class, they need to include such an option (PONV clause, for Point of Non-Viability, at the discretion of the regulator). Thus CoCo's convert if a regulator finds out at $t = 1$ that asset returns at $t = 2$ will be lower than what is compatible with a capital ratio above the CoCo's trigger value: the regulator stages an on site inspection, finds out that asset returns will be $R_L < R$ and this drop in asset value is enough to trigger conversion under the PONV clause. The regulator obtains data about R at $t = 1$ with probability zero (i.e. the event is not anticipated) and then decides whether or not to convert the CoCo's. The regulator's decision to intervene is not modeled in this paper. But his decision to convert CoCo's introduces a negative signal about asset returns even though the economic fundamentals θ remain the same.

2.4 Timing

First, let us consider the situation prior to conversion. By assumption, at $t = 0$, a fraction $\bar{e} - \bar{n}$ of agents has invested in CoCo's and a fraction $1 - \bar{e}$ has invested in equity. The remaining \bar{n} are depositors. Depositor types are only revealed at $t = 1$. The resulting type distribution is shown in Table 1.

At $t = 0$, the bank has a total of 1 unit of assets. The bank invests the entire amount in the risky asset. It also promises a fixed return $r_1 > 1$ to agents who withdraw at $t = 1$, and a stochastic

return that in the absence of runs by patient depositors equals $\tilde{r}_2 = \max \left[\frac{\bar{n} - \lambda \bar{n} r_1}{\bar{n} - \lambda \bar{n}} R, 0 \right]$. Note that this is a DD contract since $\frac{\bar{n} - \lambda \bar{n} r_1}{\bar{n} - \lambda \bar{n}} R = \frac{1 - \lambda r_1}{1 - \lambda} R$. Define n as the proportion of agents who withdraw at $t = 1$. Since impatient agents always withdraw at $t = 1$, $n \geq \lambda \bar{n}$. And because the CoCo holders and equity holders cannot withdraw early, we also have $n \leq \bar{n}$.

At $t = 1$, before agents can act, the regulator comes in and decides whether to convert CoCo's or not. If conversion occurs, the return must have been found to be some $R_L < R$. Without conversion, no information is revealed. Note that depositors' return at $t = 2$ is scaled by R , i.e. when the asset return is found to be $R_L < R$, the interest rate on deposits will change accordingly. Effectively, the depositors have a variable-rate contract with the bank. This still preserves the risk-sharing feature of Diamond-Dybvig, which concerns not so much the interest rate risk but the risk that there will not be a return at all.

Also at $t = 1$, depositor types are revealed. The bank gives $r_1 > 1$ to agents/depositors withdrawing at this time as long as it is able to do so. To this end, the bank must liquidate part of the amount invested in the risky asset. This means that the bank can only serve at most $n = \frac{1}{r_1}$ agents at $t = 1$. The period two payoffs to the depositors in the no-conversion case are summarized in Table 2. Depositors who wait until $t = 2$ will receive a return $r_D = \left(\frac{1 - \lambda r_1}{1 - \lambda} \right) R$ with probability $p(\theta)$ as long as the bank is in fact able to pay this out. CoCo holders, because they are junior to depositors, will receive amounts only once all the depositors have been served.

The bank will be able to pay out something as long as $n < \frac{1}{r_1}$, as in GP (see Eqn. 2 below), that is as long as it survives into period 2 at all. If n stays at its minimum value $\lambda \bar{n}$, i.e. only impatient depositors withdraw early, depositors will receive r_D with probability $p(\theta)$ and 0 with probability $1 - p(\theta)$, as in GP. As before, CoCo holders are junior claimants so they receive pay outs only when the remaining $(1 - \lambda) \bar{n}$ depositors have been served in period 2. With probability $p(\theta)$, equity and CoCo holders will then collectively receive $R(1 - \bar{n})$. How that surplus is divided between them depends on CoCo pricing and corresponding CoCo returns. With probability $1 - p(\theta)$, the return (to creditors and equity holders alike) will be zero.

But for n in between $\lambda \bar{n}$ and $\frac{1}{r_1}$ the outcomes are different from what they are in GP because there

Table 2: Time-dependent payoffs to each depositor

Withdrawal in	if $n < \lambda \bar{n}$	if $\lambda \bar{n} < n < \lambda \bar{n} + \frac{e'}{r_1}$	if $\lambda \bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1}$	if $n \geq \frac{1}{r_1}$
$t = 1$	r_1	r_1	r_1	$\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$
$t = 2$	$r_D = \frac{1-\lambda r_1}{1-\lambda} R$	r_D	$\begin{cases} \frac{1-nr_1}{1-(\lambda \bar{n} + \frac{e'}{r_1})r_1} r_D & \text{w.p. } p(\theta) \\ 0 & \text{w.p. } 1 - p(\theta) \end{cases}$	0

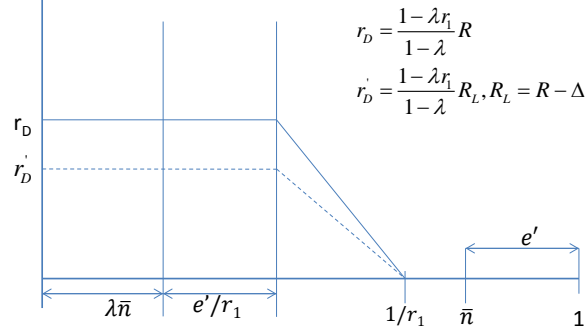
now is both junior debt (the CoCo's before conversion) and straight equity. For depositors it does not matter how these losses are allocated over junior claimants, so let us define e' as the measure of all junior creditors plus equity holders: $e' = 1 - \bar{n}$.⁴ As long as there is any capital left, junior debtors and equity holders will absorb losses before depositor pay outs are affected, so, contrary to GP, depositors still receive r_D in that region. The results differ from GP because in their set up there is no capital, GP banks have a leverage ratio equal to 1. Note that the region where money paid out to early runners eats into equity returns (because the assets generating those returns have to be liquidated) is shorter than e' because runners get paid out $r_1 > 1$ (see Figure 2 below). Once equity and junior debt claims have evaporated ($n > \lambda \bar{n} + \frac{e'}{r_1}$), depositors will increasingly see their pay out shrink too until the bank has to liquidate all assets when n reaches $\frac{1}{r_1}$ so that the bank does not survive into period 2. Figure 2 below shows the various regions and the corresponding good state of nature period 2 returns.

If the good state of nature return on the risky asset (R) turns out to be lower, say $R_L = R - \Delta < R$, the pay out schedule to period 2 depositors shifts down to the slotted line in Figure 2. We return to this below.

Throughout we are assuming that $\bar{n} > \frac{1}{r_1}$. If there is a relatively small measure of depositors ($\bar{n} \leq \frac{1}{r_1}$), then depositors know that if they all stage a run, all of them will receive r_1 . But since the incentive compatibility constraint $r_1 > p(\theta) \frac{1-\lambda r_1}{1-\lambda} R$ holds, only the impatient depositors will withdraw at $t = 1$, and there will be no run (in the sense that patient depositors also withdraw

⁴We return to the allocation of losses over junior creditors and equity holders in section 6 when we discuss CoCo design, the wealth transfers that take place upon conversion between CoCo holders and equity holders, and the incentives these transfers imply for equity holders before conversion.

Figure 2: Depositor returns at $t = 2$



early). This simply says that adequately capitalized banks ($e' > 1 - \frac{1}{r_1}$) are in no danger of a run. We will not consider this case any further.

At this point it is useful to define $v(\theta, n)$ as the difference in utility of waiting versus running for given values of θ and n , comparable to GP equation 3:

$$v(\theta, n) = \begin{cases} p(\theta)u\left(\frac{1 - nr_1}{1 - (\lambda\bar{n} + e'/r_1)r_1} r_D\right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1} \\ 0 - u(r_1)\frac{1}{nr_1} & \text{if } \frac{1}{r_1} < n < \bar{n} \end{cases}$$

2.5 Probability of a Bank Run: DD in a Global Games Framework

From Table 2, one can see that n is of primary importance in the payoff of an agent. In the DD paper, n is either only the impatient depositors, or all of the depositors (multiple equilibria). This is because they have nothing to coordinate on except for sunspots or bad expectations. Goldstein and Pauzner (2005) recast the DD bank run problem in a global games framework and in doing so obtain unique Bayesian equilibria with well defined probabilities tied to fundamentals. We follow their approach in this paper. In the global games framework, depositors obtain private and imprecise information about the economic indicator θ . In particular, at $t = 1$, each depositor obtains a private signal θ_i uniformly distributed along $[\theta - \varepsilon, \theta + \varepsilon]$, where the distribution is known to all. Clearly θ_i depends on the realization of θ . Thus depositors know that the true

value of fundamentals is at most ε away from their own signal. Depositors' decisions crucially depend on their draw of θ_i and on what they can deduce from that draw on the likely signals other depositors must have received and what they are therefore likely to do.

There are two extreme regions where depositors' decisions do not depend on what other agents do. First one can define a $\underline{\theta} = \underline{\theta}$ below which a patient depositor always finds it optimal to run even if all other patient depositors were to wait. This assumption translates into $n = \lambda \bar{n}$. Thus $\underline{\theta}$ solves the equation $u(r_1) = p(\underline{\theta})u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$. GP call the region $[0, \underline{\theta})$ the *lower dominance region*. There are always feasible values in the lower dominance region such that all signals will fall into that region if $\underline{\theta} > 2\varepsilon$; for this to obtain it is sufficient if $\underline{\theta}(1) > 2\varepsilon$ since $\underline{\theta}(r_1)$ is increasing in r_1 as can be seen by differentiating the implicit equation defining $\underline{\theta}$. $\underline{\theta}(1) > 2\varepsilon$ in turn can be rewritten as $p^{-1}\left(\frac{u(1)}{u(R)}\right) > 2\varepsilon$, which shows that ε can always be chosen small enough for the lower dominance region to be non-empty.

One can similarly define a $\bar{\theta}$ above which a patient depositor finds it optimal to wait even if all other patient agents were to run (in GP's terminology the upper dominance region). GP assume that in the region $(\bar{\theta}, 1]$, the investment is certain to yield R ($p(\theta) = 1$ for $\theta > \bar{\theta}$). Then it is never optimal to run since $R > r_1$. Alternatively one can assume a Central Bank standing ready to provide liquidity in a run for high enough θ since in that case the bank is clearly solvent. Either way, we follow GP in postulating the existence of such an upper dominance region. Since ε can be chosen arbitrarily small, we can also safely assume that it is possible that all draws fall into the upper dominance region, which requires $\bar{\theta} < 1 - 2\varepsilon$.

Within the region $[\underline{\theta}, \bar{\theta}]$, depositors must rely on equilibrium behavior of other depositors receiving nearby signals, which in turn depends on their nearby signals, and so on; continuity requires that behavior smoothly pastes to the behavior in the extreme regions. Following GP, one can prove that the unique equilibrium strategy is a switching strategy in which patient depositors run if they receive a signal $\theta_i \leq \theta^*$ and wait otherwise⁵. θ^* is defined such that a depositor receiving a signal θ^* is indifferent between waiting and running at $t = 1$ over all possible outcomes of other

⁵Since the proof in our set up is almost identical to the corresponding proof in GP, we refer to GP for a detailed proof.

depositors' behavior:

$$\int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta = \theta^*) u \left(\frac{1 - nr_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1})r_1} r_D \right) - u(r_1) \right] dn - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = \text{(D)}$$

$$\Rightarrow \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta = \theta^*) u \left(\frac{1 - nr_1}{1 - (\lambda\bar{n} + e'/r_1)r_1} \frac{1 - \lambda r_1}{1 - \lambda} R \right) - u(r_1) \right] dn - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0$$

where Eqn. 1 defines θ^* implicitly and is formed from the payoffs described in Table 2 and the function v defined at the end of section 2.4.⁶ Because the depositors obtain signals θ_i from a uniform distribution around θ and θ is itself uniformly distributed over $[0, 1]$, a higher θ^* means depositors run in a larger set of signals. For small ε , θ^* can be interpreted as the probability of a bank run. Also, each θ^* corresponds to an n which is the measure of the number of runners at $t = 1$ for given value of θ . This is⁷

$$n = \lambda\bar{n} + (1 - \lambda)\bar{n} \left[\frac{1}{2} + \frac{\theta^* - \theta}{2\varepsilon} \right] \quad (2)$$

for $\theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon$. $n = \bar{n}$ for $\theta < \theta^* - \varepsilon$ and $n = \lambda\bar{n}$ for $\theta > \theta^* + \varepsilon$.

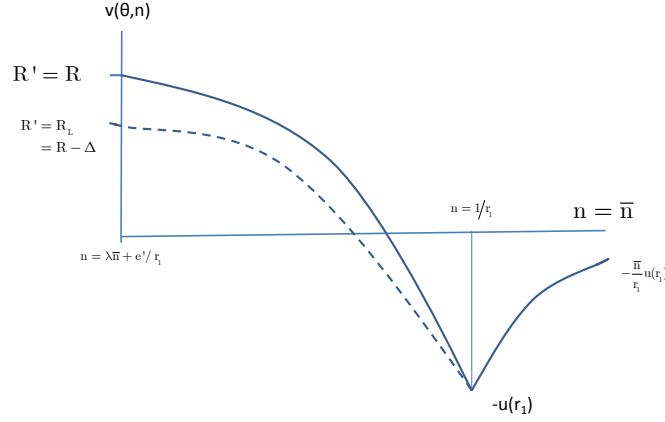
3 Effect of CoCo conversion on the probability of a run θ^*

Consider now the case when the regulator finds out that the return will be low. While θ^* depends on r_1 , it also depends on R and \bar{n} . As mentioned in Section 2.3, we introduce the regulator action at $t = 1$, before depositors can act. In the absence of CoCo conversion, depositors and other investors believe that the return of the risky asset is R with probability $p(\theta)$ and 0 with probability $1 - p(\theta)$. But when the regulator forces CoCo's to convert, a signal is given that the return of the risky asset is now some $R_L < R$, without an accompanying change in the state of fundamentals θ . The impact of a lower R on period 2 pay outs can be seen in Figure 2 (the shift from the solid to the slotted

⁶Eqn. 1 builds on the fact that θ is uniformly distributed. Since n is linear in its arguments, n must also be uniformly distributed. The expression also assumes that $p(\theta) \approx p(\theta^*)$ for ε small enough, following GP.

⁷This is similar to the GP equilibrium n scaled down by \bar{n} .

Figure 3: Utility differential of waiting versus early withdrawal for different values of R



line) and Figure 3 below. In Figure 3 we show the impact on the utility difference between waiting and early withdrawal for given θ and n . From the diagram it should be clear that once integrated over the entire range of n , the utility differential shifts against waiting, so the indifference point in state space, θ^* , will have to shift up to restore balance. So the threshold θ^* increases when the return of the risky asset is reduced to R_L .

To prove this formally, we compute the threshold θ^* from the function that implicitly defines it . This function was introduced in Section 2.5 as Eqn. 1. For convenience let us call this function as

$$\hat{f}(\theta^*, r_1, R)$$

$$\hat{f}(\theta^*, r_1, R) = \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta^*, n)) u \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} \frac{1-\lambda r_1}{1-\lambda} R \right) - u(r_1) \right] dn - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0,$$

where θ was written as a function of n 's intermediate value (away from $n = \lambda\bar{n}$ or $n = \bar{n}$), and θ is assumed to be within ε -distance of θ^* . That is, $\theta = \theta^* + \varepsilon \left[1 - \frac{2}{1-\lambda} \left(\frac{n}{\bar{n}} - \lambda \right) \right]$ (see Eqn. 2). At $\theta = \theta^*$, a patient agent is indifferent between waiting or running, by definition of θ^* .

Note that since $\hat{f}(\cdot)$ is increasing in both R and θ , so in order to keep $\hat{f}(\cdot) = 0$, a decrease in R must be compensated by an increase in θ . From the implicit function theorem,

$$\frac{\partial \theta^*}{\partial R} = - \left(\frac{\partial \hat{f}}{\partial R} \right) \div \left(\frac{\partial \hat{f}}{\partial \theta^*} \right)$$

It is easy to see that $\frac{\partial \hat{f}}{\partial \theta^*} > 0$: the dependence runs completely through $\theta(n, \theta^*)$, while $\theta(n, \theta^*)$ rises in θ^* and \hat{f} rises in θ because $p'(\cdot) > 0$ by construction. Now

$$\begin{aligned} \frac{\partial \hat{f}}{\partial R} &= \int_{n=\lambda\bar{n}+\frac{\epsilon'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta^*, n)) \frac{\partial u \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{\epsilon'}{r_1})r_1} \frac{1-\lambda r_1}{1-\lambda} R \right)}{\partial R} \right] dn \\ &= \int_{n=\lambda\bar{n}+\frac{\epsilon'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta^*, n)) u'(\cdot) \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{\epsilon'}{r_1})r_1} \frac{1-\lambda r_1}{1-\lambda} \right) \right] dn \\ &> 0 \end{aligned}$$

since $\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{\epsilon'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right)$ is positive over the entire interval of integration. And with $\hat{f}_R > 0$, $\hat{f}_{\theta^*} > 0$ and $\frac{\partial \theta^*}{\partial R} = -\hat{f}_R / \hat{f}_{\theta^*}$, Proposition 1 below follows:

Proposition 1. θ^* is decreasing in R : $\frac{\partial \theta^*}{\partial R} < 0$ for all values of R

As a consequence, a negative signal on asset returns will lead to a higher run probability θ^* . This holds for any negative signal that the depositors obtain about the return R . CoCo conversion delivers a signal which is always negative because the conversion, which only takes place after adverse events leading to a lower capitalization, is itself the signal. Upon learning this news, each depositor will expect a lower differential payoff than its value before conversion, since $R_L < R$ (see Figure 2). If for return R the depositors are just indifferent between running and waiting for a given θ^* , then for return $R_L < R$ it must be that the depositors would prefer to run for the same value of θ^* . In order to make the depositors once again indifferent between running and waiting for return $R_L < R$, they must obtain a higher signal about the fundamentals, that threshold value will go up: $\theta_L^* > \theta^*$ at the point of indifference, which is what Proposition 1 says. But since depositors' θ_i are uniformly distributed between $[0, 1]$, a greater measure of them will have $\theta_i < \theta_L^*$, which implies a higher probability of a run. Note that the increase in θ^* also results in an increase in n for given value of ϵ and θ .

Proposition 1 also has an important corollary on the impact of the trigger level of a CoCo. The

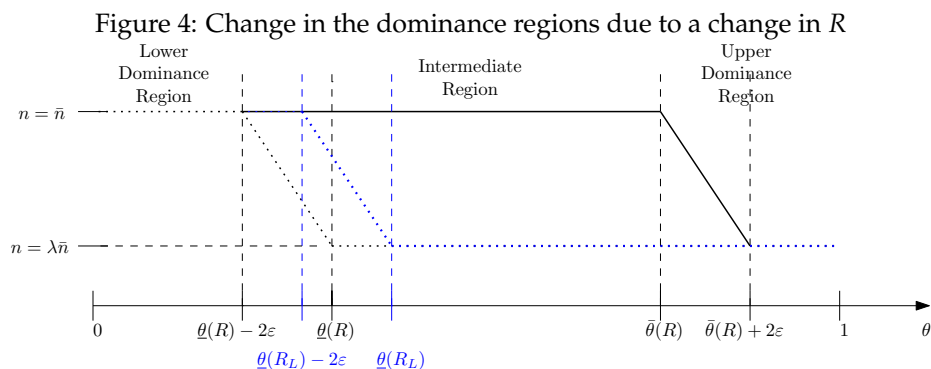
basic point is that if a conversion is triggered at a low trigger level, the implied asset quality signal is larger (i.e. more negative) than if the CoCo would have been triggered earlier, i.e. at a higher capital ratio. Define the trigger ratio τ as the Capital to Asset ratio (CAR) threshold below which a regulator will force conversion of the CoCo containing that trigger ratio upon finding out that the actual CAR has fallen below τ . And define τ_H and τ_L as the trigger level of a high trigger level H-CoCo and a low trigger level L-CoCo. Also define CAR_0 as the CAR thought to apply before the regulator's on site inspection revealed a shortfall. It should be clear that conversion of the L-CoCo gives a signal that is more negative than conversion of an H-CoCo would have given, by $CAR_0 \cdot (\tau_H - \tau_L)$ in absolute terms. Corollary 2 then follows immediately from Proposition 1:

Corollary 2. *Conversion of a CoCo with a high trigger level will lead to a smaller increase in run probability than conversion of a low trigger CoCo*

Define θ_H^* and θ_L^* as the run probability that will obtain after conversion of a high (H) respectively a low (L) trigger CoCo. Direct application of Proposition 1 with the definitions just introduced shows that the following holds (exactly, since the derivative is positive for all R so we can apply the Mean Value theorem):

$$\theta_L^* - \theta_H^* = - \left(\frac{\partial \theta^*}{\partial R} \right) \cdot (\tau_H - \tau_L) \cdot CAR_0 > 0$$

This result suggests that the BIS is right to insist on sufficiently high trigger levels before CoCo's are accepted as part of T1. According to the Basel committee (BCBS 2011), CoCo's will be either Tier 2 (T2) or Additional Tier 1 (AT1) capital, depending on their trigger ratio: a trigger above 5.125% satisfies the *going concern* requirement for AT1 and thus allows classification as AT1. Lower triggers lead to a classification as *gone concern instruments* and consequently to a T2 status. The impact of a conversion is that the CoCo holders change status from being subordinated debtors to residual claimants. This lowers the issuing bank's leverage ratio, and increases its common equity tier 1 (CET1) capitalization. If the CoCo design did not satisfy Tier 1 (T1) requirements (for example because of a trigger ratio that is too low to satisfy the going concern requirement,



conversion will increase the bank's overall T1 capital requirement also.

It is also worth noting that a change from R to R_L alters the dominance regions. Because the supremum for the lower dominance region is determined by the equation $u(r_1) = p(\underline{\theta})u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$, a change from R to R_L necessarily increases $\underline{\theta}$. Also, the infimum of the upper dominance region should not increase but may decline because if a minimum of $\bar{\theta}$ ensures that R will be obtained with certainty, then there must at least as many θ -values that will ensure R_L will be obtained with certainty. This means that the post-conversion $\bar{\theta}$ must be no lower than the pre-conversion one. Figure 4 shows the shift in the dominance regions and the effect on n .

To recapitulate: a CoCo conversion will increase a bank's equity base or at least its T1 capital ratio, depending on the type of CoCo, but the conversion nevertheless will increase the probability that depositors will stage a run. The key intuitive point is that depositors were anyhow senior to the CoCo holders, so conversion does not affect their position in situations of distress but does convey a negative signal on asset returns. Conversion sets of wealth and risk transfers between classes of creditors who are all junior to depositors so all depositors will pay attention to is the negative signal.

4 CoCo design and run probabilities after conversion

Until now we have left unspecified what specifically happens after conversion. What happens after banks fall below the trigger value depends on the type of CoCo issued. For CoCo's to qualify as capital at all (whether that be T2 or AT1, see the preceding section for a detailed discussion), they need to include a so called Point of No Viability trigger, i.e. the possibility for the regulator to enforce conversion if the regulator decided that viability is threatened. Currently used CoCo designs fall into 3 distinct types⁸.

First are convert-to-equity (CE) CoCo's such as those issued for example by Lloyds recently. These CoCo's completely convert to equity at some conversion rate ψ . Most commentators and some academics (cf Martynova and Perotti (2012)) have this type of CoCo design in mind when discussing CoCo's in general.

Next are principal writedown (PWD) CoCo's. Upon breaching the trigger value, these CoCo's are partially or entirely written down. In case of partial write down, the remaining part effectively turns into subordinated debt. The effect is also a reduction in the issuing bank's leverage ratio, but without issuing new shares of equity. Japan's Mizuho Financial Group⁹ has most recently issued such CoCo's. The impact of conversion on the various capital ratios is as discussed in the preceding section.

Finally there are also principal writedown CoCo's with cash outlays (CASH). Similar to the PWD CoCo's, CASH CoCo's are also (partially) written off upon the bank's breach of the trigger value. The remaining value is paid out in cash. Notably, Rabobank of the Netherlands has issued this type of CoCo. CASH CoCo's may or may not reduce the leverage ratio of a converting bank depending on the numerical value of the parameters (percentage writedown). The write down works in the right direction, but the cash pay out undermines that effect. Since there is most likely no capital requirement against highly liquid assets, CASH CoCo's do not reduce risk-weighted assets (RWA) upon conversion and so will improve the RWA-based CET1 capital ratio and the

⁸cf van den Berg et al. (2014) and Avdjiev et al. (2013) for an extensive description of all CoCo's issued up until 2013 and 2014 respectively

⁹Financial Times: March 26, 2014

Table 3: Depositor payoffs: regulatory forbearance (undisclosed low returns R_L)

	If $n < \frac{1}{r_1}$	If $n \geq \frac{1}{r_1}$
$t = 1$	r_1	$\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$
$t = 2$	$\begin{cases} \left(\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e}{r_1})r_1} \right) \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}$	0

overall T1 ratio since the CASH CoCo presumably does not qualify as T1 before conversion. Of course if the bank needs to sell risk assets to raise the cash to pay out on the CASH CoCo's, there is an impact on RWA and the impact on capital ratios is more complex. Also, having to sell risk assets in the middle of a distress situation may trigger firesales with corresponding problems (see Brunnermeier et al. (2013) for a discussion of firesale amplification cycles). A surprising feature of this type of CoCo design is indeed that it implies a cash call in times of distress. Because of the automatic cash pay out, this type of CoCo's probably does not qualify as either AT1 or T2 capital before conversion.

In this section, we examine the impact of CoCo design on the probability of a bank run after conversion and on the equity position of the bank if partial runs do occur.

4.1 Baseline Case: Regulatory Forbearance (RF)

As a benchmark, we consider the case where the regulator finds out that returns will be low but decides to not publicize this finding. The CoCo's do not convert. Depositors base their behavior on the belief that in good states of nature returns are R but in fact they will be R_L . Table 3 shows the payoffs to depositors.

Tables 2 and 3 are almost identical. However, in Table 3, we show the case when returns are found out to be low. But if depositors do not know returns will be low, the differential payoff function

remains the same as in Eqn. 3 .

$$v_L = \begin{cases} p(\theta)u \left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right) R \right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n} \end{cases} \quad (3)$$

The corresponding implicit function that determines θ_{rf}^* is given by Eqn. 4.

$$\begin{aligned} \hat{f}(\theta_{rf}^*, r_1, R) &= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta_{rf}^*, n))u \left(\left[\frac{1-nr_1}{1-(\lambda\bar{n}+\frac{e'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right) R \right) - u(r_1) \right] dn \\ &\quad - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned} \quad (4)$$

Obviously, since the derivations of θ_{rf}^* are based on the same set of beliefs as in our base case without bad news, $\theta_{rf}^* = \theta^*$. Depositors do not know that R has fallen to R_L , so the run probability θ^* is not affected. In the event that $n < \frac{1}{r_1}$ at $t = 1$, then

$$(\bar{n} - n) \left[\frac{1 - nr_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1})r_1} \right] \left(\frac{1 - \lambda r_1}{1 - \lambda} \right) R_L$$

will be given to the remaining depositors who didn't run at $t = 1$ (this amounts to $\bar{n} - n$ depositors), while what remains of the asset base

$$R_L \left[(1 - nr_1) - \frac{(\bar{n} - n)(1 - nr_1)(1 - \lambda r_1)}{(1 - (\lambda\bar{n} + \frac{e'}{r_1})r_1)(1 - \lambda)} \right]$$

will be used to first pay out the junior CoCo holders (who collectively have $\bar{e} - \bar{n}$ worth of claims that earn a return r_{coco} per unit¹⁰). Finally, anything that remains after that will go to equity holders.

¹⁰Here r_{coco} is an arbitrary return to CoCo holders. In this paper we are taking this return as a given, as we do not get into the pricing of CoCos.

Table 4: Depositor payoffs after CE CoCo's conversion

	If $\lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1}$	If $n \geq \frac{1}{r_1}$
$t = 1$	r_1	$\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$
$t = 2$	$\begin{cases} \left(\frac{1-nr_1}{1-(\lambda\bar{n} + \frac{e'}{r_1})r_1} \right) \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}$	0

Thus, assuming that $R_L \left[(1 - nr_1) - \frac{(\bar{n} - n)(1 - nr_1)(1 - \lambda r_1)}{(1 - (\lambda\bar{n} + \frac{e'}{r_1})r_1)(1 - \lambda)} \right] > 0$, the remaining equity base under regulatory forbearance (E_{rf}) will be

$$E_{rf} = \max \left\{ R_L \left[(1 - nr_1) - \frac{(\bar{n} - n)(1 - nr_1)(1 - \lambda r_1)}{(1 - (\lambda\bar{n} + \frac{e'}{r_1})r_1)(1 - \lambda)} \right] - r_{coco}(\bar{e} - \bar{n}), 0 \right\}$$

Under regulatory forbearance, there is no way to reduce a bank's liabilities, so any negative asset development immediately eats into equity (and CoCo's once limited liability becomes a binding constraint on the decline in equity). We will come back to this point later.

4.2 Convert-to-equity (CE) CoCo's

Consider now the case where the regulator does force conversion, CE CoCo holders turn into equity holders upon conversion, and therefore, forfeit the right to receive the amount up to $\bar{e} - \bar{n}$ but become entitled to a share in any residual income. In other words, only $\bar{n} - n$ agents have to be paid out the promised amount at $t = 2$. Table 4 shows the resulting payoffs to depositors. The differential payoff function used by depositors is different now, since depositors do receive the negative signal associated with CoCo conversion. Eqn. 5 is different from Eqn. 3, R is replaced by

the lower value R_L :

$$v_{ce} = \begin{cases} p(\theta)u \left(\left[\frac{1-nr_1}{1-(\lambda\bar{n} + \frac{e'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L \right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n} \end{cases} \quad (5)$$

As before, we can compute the threshold run value of the economic fundamental for a CE CoCo implicitly. Denote by θ_{ce}^* the probability of a run for the CE case. As before, the equation that implicitly defines θ_{ce}^* is given by Eqn. 6.

$$\begin{aligned} \hat{f}_{ce}(\theta_{ce}^*, r_1, R_L) &= \int_{n=\lambda\bar{n} + \frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta_{ce}^*, n))u \left(\left[\frac{1-nr_1}{1-(\lambda\bar{n} + \frac{e'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L \right) - u(r_1) \right] d\theta \\ &\quad - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned} \quad (6)$$

Then application of Proposition 1 immediately shows that $\theta_{ce}^* > \theta_{rf}^*$. This highlights the bind regulators are in when they find out they should force CoCo's to convert. The negative signal that conveys to depositors actually increases financial fragility.

On the other hand, converting CE CoCo's increases the equity at $t = 2$ relative to the RF case. This is clear because when CE CoCo's are converted, the CoCo holders do not have to be paid out at $t = 2$ anymore. Let n_{ce} denote the number of runners implied by the probability of bank run θ_{ce}^* . We can actually see the beneficial effect of a CoCo conversion, because provided that θ_{ce}^* yields $n_{ce} < \frac{1}{r_1}$, the bank survives until $t = 2$ with more capital as CoCo holders are no longer creditors. We can denote by E_{ce} the resulting equity upon conversion of the CE CoCo's.

$$E_{ce} = \max \left\{ R_L \left[(1 - n_{ce}r_1) - \frac{(\bar{n} - n_{ce})(1 - n_{ce}r_1)(1 - \lambda r_1)}{\left(1 - (\lambda\bar{n} + \frac{e'}{r_1})r_1\right)(1 - \lambda)} \right], 0 \right\} \quad (7)$$

From Section 4.1, since $\theta_{rf}^* < \theta_{ce}^*$, it must also be true that $n_{rf} < n_{ce}$. E_{ce} differs from E_{rf} by the difference between n_{rf} and n_{ce} , and also by the amount that must be paid to the CoCo holders

(which disappears in the conversion of CE CoCo's). Note that

$$\begin{aligned} E_{ce} - E_{rf} &= r_{coco} (\bar{e} - \bar{n}) + R_L (n_{rf} - n_{ce}) + \left[(n_{ce} - n_{rf}) (1 + \bar{n}r_1) + (\bar{n}_{rf}^2 - \bar{n}_{ce}^2) \right] \Gamma R_L \\ &= r_{coco} (\bar{e} - \bar{n}) + R_L (n_{ce} - n_{rf}) (\Gamma (1 + \bar{n}r_1) - 1) + (\bar{n}_{rf}^2 - \bar{n}_{ce}^2) \Gamma R_L \end{aligned}$$

$n_{ce} - n_{RF} > 0$, and $\Gamma(1 + \bar{n}r_1) - 1 = \frac{(1-\lambda r_1)(1+\bar{n}r_1)}{(1-(\lambda\bar{n}+\frac{e'}{r_1})r_1)(1-\lambda)} - 1 = \frac{(1-\lambda r_1)(1+\bar{n}r_1)}{\bar{n}(1-\lambda r_1)(1-\lambda)} - 1 = \frac{(1+\bar{n}r_1)}{\bar{n}(1-\lambda)} - 1 > 0$, using the definition of e' , so up to a first-order approximation (ignoring the quadratic terms in n), the conversion indeed improves the equity base of the bank if at least it survives into the good state of nature.

Proposition 3. *Iff $n_{ce} < \frac{1}{r_1}$ (i.e. the bank survives period 1), conversion of CE CoCo's improves the bank's equity position at $t = 2$ as the bank is able to eliminate up to $r_{coco} (\bar{e} - \bar{n})$ worth of liabilities.*

This result points to an incentive for regulators to actually force conversion once they find out about lower returns R_L . The regulator does face conflicting incentives when finding out about R_L . On the one hand, conversion increases the probability of a run because it conveys a negative signal about asset returns. On the other hand, conversion also ensures that if runs occur, there is a probability that there will be a surviving equity base, and that it will be higher than when regulators are forbearing. Regulators thus are forced to choose between keeping fragility low at the expense of worsening the consequences of a run if it does occur, and increasing the likelihood of a run but leaving the bank better equipped to deal with one.

4.3 Principal Writedown (PWD) CoCo's

We have previously described PWD CoCo's as having a fraction written down upon conversion. Let $1 - \varphi$ denote the fraction of CoCo's that is written off when conversion occurs, so φ is the fraction that is left, where $0 \leq \varphi \leq 1$. We can then write the total creditor (i.e. non-equity) claims at $t = 2$ as $\varphi (\bar{e} - \bar{n}) + (\bar{n} - n)$, provided that the bank did not fail at $t = 1$. However, the depositors $(\bar{n} - n)$ still get paid first. Table 5 describes the payoffs to depositors in the PWD case

Table 5: Depositor payoffs after PWD CoCo's conversion

	If $\lambda\bar{n} + \frac{e'}{r_1} < n < \frac{1}{r_1}$	If $n \geq \frac{1}{r_1}$
$t = 1$	r_1	$\begin{cases} r_1 & \text{w.p. } \frac{1}{nr_1} \\ 0 & \text{w.p. } 1 - \frac{1}{nr_1} \end{cases}$
$t = 2$	$\begin{cases} \left(\frac{1-nr_1}{1-(\lambda\bar{n} + \frac{e'}{r_1})r_1} \right) \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}$	0

after conversion.

As in the CE CoCo case, the amount that each depositor would obtain is the same as that under no conversion because depositors are senior to remaining CoCo holders. Therefore the differential payoff function used by depositors here (Eqn. 5) is identical to what it is in the case of CE CoCo's.

$$v_{pwd} = \begin{cases} p(\theta)u \left(\left[\frac{1-nr_1}{1-(\lambda\bar{n} + \frac{e'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L \right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n} \end{cases} \quad (8)$$

Let θ_{pwd}^* denote the threshold level of θ for the PWD case. We can again find θ_{pwd}^* from the implicit function in Eqn. 9.

$$\begin{aligned} \hat{f}_{pwd}(\theta_{pwd}^*, r_1, R_L) &= \int_{n=\lambda\bar{n} + \frac{e'}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta_{pwd}^*, n))u \left(\left[\frac{1-nr_1}{1-(\lambda\bar{n} + \frac{e'}{r_1})r_1} \right] \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_L \right) - u(r_1) \right] dn \\ &\quad - \int_{n=\frac{1}{r_1}}^{\bar{n}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned}$$

Since the differential pay off function is the same, it follows that $\theta_{pwd}^* = \theta_{ce}^*$. This means that PWD CoCo's are not an improvement over CE CoCo's if evaluated for their impact on probability of runs after a bad signal because neither type of CoCo changes the incentives for depositors. The explanation is straightforward: PWD and CE CoCo's imply different wealth transfers between CoCo holders and equity holders, but since depositors are senior to both groups of claimants, they do not care how losses are allocated between them.

Table 6: Payout to depositors after CASH CoCo's conversion

	If $\lambda\bar{n} + \frac{e'}{r_1} - \delta < n < \frac{1}{r_1} - \delta$	If $n \geq \frac{1}{r_1} - \delta$
$t = 1$	r_1	$\begin{cases} r_1 & \text{w.p. } \frac{1}{n} \left(\frac{1}{r_1} - \delta \right) \\ 0 & \text{w.p. } 1 - \frac{1}{n} \left(\frac{1}{r_1} - \delta \right) \end{cases}$
$t = 2$	$\begin{cases} \left[\frac{1 - \delta r_1 - n r_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} \right] \left(\frac{1 - \lambda r_1}{1 - \lambda} \right) R_L & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}$	0

upon conversion, there is correspondingly less cash available to pay out in case of early withdrawals. This will affect the differential pay off function, and therefore θ^* and the corresponding expected number of runners n . Consider first the impact of paying out cash on n . The cash payout decreases the *maximum* value of n from $\frac{1}{r_1}$ to $\frac{1}{r_1} - \delta$. However, because we let the equity holders and the coco holders absorb first losses, this also means that the value of n where the amount r_D is scaled by the number of runners is also pushed back by δ (from $\lambda\bar{n} + \frac{e'}{r_1}$ to $\lambda\bar{n} + \frac{e'}{r_1} - \delta$). This means that the bounds of n change. Eqn. 10 shows the differential payoff function for the CASH case.

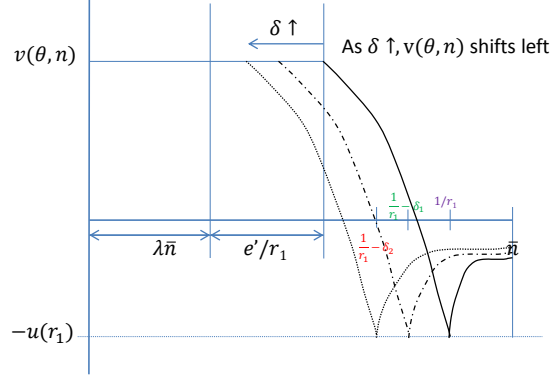
$$v_{cash} = \begin{cases} p(\theta)u \left(\left[\frac{1 - \delta r_1 - n r_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} \right] \left(\frac{1 - \lambda r_1}{1 - \lambda} \right) R_L \right) - u(r_1) & \text{if } \lambda\bar{n} + \frac{e'}{r_1} - \delta \leq n \leq \frac{1}{r_1} - \delta \\ 0 - \frac{1}{n} \left(\frac{1}{r_1} - \delta \right) u(r_1) & \text{if } \frac{1}{r_1} - \delta \leq n \leq \bar{n} \end{cases} \quad (10)$$

The equation that implicitly defines θ_{cash}^* can be formed from the differential payoff equation. This is given by Eqn. 11.

$$\begin{aligned} \hat{f}_{cash}(\theta_{cash}^*, r_1, R_L) &= \int_{n=\lambda\bar{n} + \frac{e'}{r_1} - \delta}^{\frac{1}{r_1} - \delta} \left[p(\theta(\theta_{cash}^*, n))u \left(\left[\frac{1 - \delta r_1 - n r_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} \right] \left(\frac{1 - \lambda r_1}{1 - \lambda} \right) R_L \right) - u(r_1) \right] dn \\ &\quad - \int_{n=\frac{1}{r_1} - \delta}^{\bar{n}} \frac{1}{n} \left(\frac{1}{r_1} - \delta \right) u(r_1) dn = 0 \end{aligned}$$

We can see that as $\delta \rightarrow 0$, $\theta_{cash}^* \rightarrow \theta_{pwd}^* = \theta_{ce}^*$. However, because the bounds of the integral change along with the expression within the utility function, it is difficult to be precise unless we look at the derivative of θ_{cash}^* with respect to δ . A cash payout δr_1 reduces the amount that is available

Figure 6: Differential utility for different values of δ



to depositors who wait until $t = 2$ (see the reduction in the numerator of $u(\cdot)$). However, by choosing to wait, depositors forgo receiving r_1 at $t = 1$. If n falls into the range $\frac{1}{r_1} - \delta \leq n \leq \bar{n}$, a depositor's "expected opportunity loss" is only $-\frac{1}{n} \left(\frac{1}{r_1} - \delta \right) u(r_1)$ rather than $-\frac{1}{nr_1} u(r_1)$. As such, there is less to lose by waiting if n happens to be large, but one must note as well that the range $\left[\frac{1}{r_1} - \delta, \bar{n} \right]$ rises with δ . The ambiguity arises because both the gain from waiting and the loss from waiting fall at the same time. Figure 6 illustrates the differential payoff functions for different values of δ .

In this section, we follow the earlier procedures and calculate the derivatives of θ_{cash}^* with respect to δ explicitly using the implicit function theorem. The expressions are laborious and so relegated to the Annex, but we can unambiguously sign the derivative: $\frac{\partial \theta_{cash}^*}{\partial \delta} > 0$. The impact of δ on the gain from waiting is higher than its impact on the expected opportunity loss from waiting. Thus, a higher θ_{cash}^* is needed to compensate for the impact of an increase in the cash component δ .

Proposition 5. θ_{cash}^* is increasing in δ : $\frac{\partial \theta_{cash}^*}{\partial \delta} > 0$

For the proof see the Annex. Combining Proposition 5 with our earlier results allows us to give a definitive ranking of the types of CoCos in terms of impact on probability of bank runs:

Corollary 6. For $\delta > 0$, $\theta_{rf}^* < \theta_{ce}^* = \theta_{pwd}^* < \theta_{cash}^*$

5 Contagion and Systemic Risk

5.1 Contagion

Banks may have correlated asset returns for several reasons. The most obvious one is that banks often have cross-holdings of deposits (Allen and Gale (2000)). Another is when banks invest in the same set of industries, either by intentionally herding (like in Acharya and Yorulmazer (2008, 2007); Farhi and Tirole (2012)) or as a result of their individual diversification policies as in Wagner (2010). Banks also tend to invest in similar assets as a result of conforming to regulatory requirements by institutions such as BCBS (as in Iannotta and Pennacchi (2012)). Thus, negative information about one bank may have an adverse impact on other financial institutions. This information contagion effect has been well-documented (empirically) in the literature and is not confined to the banking sector Aharony and Swary (1983, 1996); Lang and Stulz (1992). Thus, when CoCo's of one bank convert, they impose an information externality on the other banks who hold assets with returns correlated to those of the converting bank. In this section we show how this could happen.

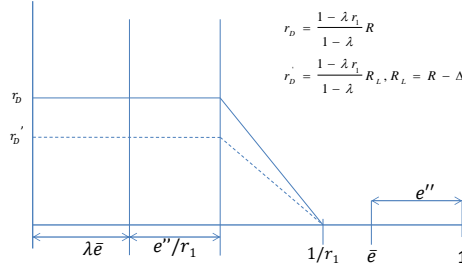
To do so we consider a two-bank system. Let Bank 1 be a CoCo-issuing bank (as discussed in Sections 4.2, 4.3 and 4.4) (at this stage, the type of CoCo doesn't matter - only the conversion does) and without loss of generality, let Bank 2 be an ordinary bank without CoCo's. Similar to Bank 1, Bank 2 also has a continuum of depositors who obtain private signals $\theta_{2i} \sim U[\theta_2 - \varepsilon, \theta_2 + \varepsilon]$, and investments in risky technology with return R_2 but no CoCo's. Table 7 summarizes the setup for the two-bank case.

As Bank 2 depositors also obtain private signals θ_{2i} , its patient depositors also decide whether to wait or to run at $t = 1$ depending on their posterior assessment of θ_2 . The decision is made by

Table 7: Summary of Bank Features: Two-Bank System

	Bank 1	Bank 2
Bank type	CoCo-issuing	ordinary
agents	continuum from $[0, 1]$	continuum from $[0, 1]$
CoCo holders	$\bar{e} - \bar{n}$	0
equity holders	$1 - \bar{e}$	$1 - \bar{e}$
impatient depositors	$\lambda \bar{n}$	$\lambda \bar{e}$
patient depositors	$(1 - \lambda) \bar{n}$	$(1 - \lambda) \bar{e}$
probability of run	θ_1^*	θ_2^*
potential returns	0 or $R_1 \in \{R_L, R\}$	0 or $R_2 = \{R_L, R\}$

Figure 7: Depositor payouts at $t = 2$ for a non-coco bank



using the differential payoff function for depositors of Bank 2, shown in Eqn. 12.

$$v_2(\theta, n) = \begin{cases} p(\theta) u \left(\left(\frac{1 - nr_1}{1 - (\lambda \bar{e} + \frac{e''}{r_1}) r_1} \right) \left(\frac{1 - \lambda r_1}{1 - \lambda} \right) R_2 \right) - u(r_1) & \text{if } \lambda \bar{e} + \frac{e''}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{e} \end{cases} \quad (12)$$

where $e'' = 1 - \bar{e}$ (See Figure 7). As before, there is only one value of θ which makes them indifferent between waiting and running. Call this value θ_2^* . As before, this can be interpreted as the probability of a run in Bank 2, and is defined implicitly by its differential payoff function where now $\lambda \bar{e} \leq n \leq \bar{e}$ because Bank 2 did not issue CoCo's.

The function that implicitly defines Bank 2's probability of a run is given by¹¹

$$\begin{aligned} \hat{f}(\theta_2^*, r_1, R) &= \int_{n=\lambda\bar{e}+\frac{e''}{r_1}}^{\frac{1}{r_1}} \left[p(\theta(\theta_2^*, n))u \left(\left(\frac{1-nr_1}{1-(\lambda\bar{e}+\frac{e''}{r_1})r_1} \right) \left(\frac{1-\lambda r_1}{1-\lambda} \right) R_2 \right) - u(r_1) \right] dn \\ &\quad - \int_{n=\frac{1}{r_1}}^{\bar{e}} \frac{1}{nr_1} u(r_1) dn = 0 \end{aligned} \quad (13)$$

We now want to determine the impact of Bank 1's CoCo conversion on Bank 2's probability of a bank run. Formally, we want to determine the sign of the derivative $\frac{\partial \theta_2^*}{\partial R_1}$ at $t = 1$. This can be written as

$$\frac{\partial \theta_2^*}{\partial R_1} = \frac{\partial \theta_2^*}{\partial R_2} \frac{\partial R_2}{\partial R_1} \quad (14)$$

where the first term is the impact of a change in the return of Bank 2 on the Bank 2's run probability. From Proposition 1, it is clear that $\frac{\partial \theta_2^*}{\partial R_2} < 0$. The sign of $\frac{\partial R_2}{\partial R_1}$ of course depends on the correlation of R_2 and R_1 . If they are positively correlated, $\frac{\partial R_2}{\partial R_1} > 0$. If not, then $\frac{\partial R_2}{\partial R_1} = 0$. Any information about R_1 (and therefore R_2) is revealed only when CoCo's convert. Otherwise, no information is revealed. Thus, we have that in the event of a CoCo conversion and correlated asset returns, $\frac{\partial \theta_2^*}{\partial R_1} < 0$.

Proposition 7. *If bank returns are correlated, CoCo conversion in one bank leads to an increase in the run probability of another bank. This is true regardless of the type of CoCo issued by the converting bank.*

We have mentioned in Section 2 that when \bar{n} is small ($\bar{n} < \frac{1}{r_1}$), depositors know that they will all be served at $t = 1$ if they all withdraw. In this case only the impatient depositors withdraw, and all the patient depositors wait until $t = 2$. However, this small \bar{n} does not preclude the possibility that the regulator finds it necessary to force conversion of CoCos.

From Proposition 7, the knowledge of Bank 1's conversion leads Bank 2 depositors to have a higher required threshold $\theta_2^{**} > \theta_2^*$. This increases the proportion of depositors who obtain signals that are lower than the new threshold. Thus while conversion in Bank 1 may not cause a run in Bank 1,

¹¹This, along with Eqn. 12 is the θ^* derivation in GP but scaled by \bar{e} (no CoCo's).

it raises the probability of runs in Bank 2. Moreover, it may even cause full runs in Bank 2 because $n \in [\lambda\bar{e}, \bar{e}] \supseteq [\lambda\bar{n}, \bar{n}]$ such that when θ_2^{**} is high enough, the associated n_2 exceeds $\frac{1}{r_1}$.

Corollary 8. *CoCo conversion of Bank 1 leads to a higher probability of bank runs in Bank 2 (and possibly even failure), even if Bank 1 has small \bar{n} .*

5.2 Systemic Risk

From the above discussion, it is only a small step to show that CoCo conversion raises systemic risk. In general, systemic risk can be described as a situation where the banks fail at the same time, or if the failure of one bank spreads to other banks. While banks are not compelled under Basel 3 to issue CoCo's, an increasing number of banks have been issuing them¹². It is therefore natural to examine the impact on systemic risk of having CoCo's in the banking system. Since systemic risk is the risk that banks fail jointly, and since in this paper, we define bank failure as being due to bank runs, we quantify systemic risk to be the product of each bank's probability of a run, as in Ahnert and Georg (2012).

It is straightforward to show that systemic risk rises when CoCos convert, regardless of the type of CoCo issued. To do this, we still work with the two-bank case, as with the discussion on contagion. From Proposition 7, $\frac{\partial\theta_2^*}{\partial R_1} > 0$. Let us call the raised value of θ_2^* due to a drop in the returns of Bank 1 as θ_2^{**} . Thus, $\theta_2^{**} > \theta_2^*$.

As a benchmark case, consider what would happen when the regulator exercises forbearance and doesn't convert CoCos of Bank 1 despite knowing that returns will be low. Then, Bank 2 depositors would not obtain any signal about R , and they would not be affected. Systemic risk would then be $\theta_{rf}^* \cdot \theta_2^*$.

Next, consider what would happen when the regulator decides to convert the CoCos of Bank 1. Bank 2 depositors would infer that Bank 1 returns would be low. This has an impact on the returns of Bank 2, which would also be lower. As a result, Proposition 7 applies and so the probability of

¹²Financial Times, May 26, 2014

bank runs in Bank 2 is now θ_2^{**} . Systemic risk would then be $\theta_{convert}^* \cdot \theta_2^{**}$, where we use $\theta_{convert}^*$ to refer to the heightened θ^* from any kind of converted CoCo. Note that we now have

$$\theta_{convert}^* \cdot \theta_2^{**} \geq \theta_{rf}^* \cdot \theta_2^*$$

The contagion effect becomes more obvious when we increase the number of banks. Suppose we have n banks, where Bank 1 is a CoCo bank, while the remaining $n - 1$ banks are same-sized regular banks (one could think there are $n - 1$ clones). Suppose also that bank asset returns are correlated throughout the financial system. Systemic risk is then the product of the probability of bank runs of all n banks:

$$\theta_1^* \times \theta_2^* \times \dots \times \theta_n^*$$

Systemic risk under regulatory forbearance would be

$$\theta_{rf}^* \cdot (\theta_2^*)^{n-1}$$

While systemic risk under a watchful regulator would be

$$\theta_{convert}^* \cdot (\theta_2^{**})^{n-1}$$

And as before

$$\theta_{convert}^* \cdot (\theta_2^{**})^{n-1} > \theta_{rf}^* \cdot (\theta_2^*)^{n-1}$$

As $n \rightarrow \infty$, the difference becomes larger because $\frac{\theta_2^{**}}{\theta_2^*} > 1$, which rises with n , while Bank 1's θ s are not affected by n .

Proposition 9. *When the regulator is forbearing, systemic risk due to bank runs at $t = 1$ perversely remains low. On the other hand, when the regulator is not forbearing and forces CoCo conversion in one*

bank, systemic risk rises. Moreover, as the number of banks increase, systemic risk rises more.

6 Wealth transfer effects of CoCo conversions

In the discussion so far we have almost completely focused on how conversion and in particular the signal that it conveys changes the incentives that depositors face. Conversion does not affect depositors directly but their run incentives change nevertheless because the act of conversion constitutes a negative signal on asset quality. But conversion in itself basically reallocates losses over CoCo holders and equity holders, both of whom are junior to depositors, hence the lack of a direct impact (other than the consequences of the signal conversion gives) on deposit holders. But there generally will be a very direct impact on the junior claim holders, be they the CoCo holder or equity holders. Which way that goes depends critically on the CoCo design, to which we turn in this section. We measure wealth transfers by comparing what each original equity holder would obtain after conversion of different types of CoCo's to a benchmark case where the bad signal is given, but no conversion takes place. Note that only outcomes in the good state of nature (occurring with probability $p(\theta)$) are of interest because in the bad state of nature, occurring with probability $1 - p(\theta)$, equity holders and CoCo holders alike will be wiped out anyhow whether there is conversion or not and whatever the CoCo design.

6.1 Benchmark case

To examine whether wealth transfers occur, we need to set up a benchmark case as defined: the signal on deterioration of asset quality has gone out, so returns have been found out to be R_L , but cocos have not converted. The benchmark is chosen this way to isolate the *direct* impact of CoCo conversion, separate from the indirect impact through the asset quality signal since that signal will go out under all CoCo designs, and comparison between different designs is the focus of this section. We have seen in Sections 3 and 4 that for CE and PWD CoCo's, θ^* responds to the asset quality signal only, so in the benchmark case θ^* rises to the same level, call it for convenience' sake

$\theta_{benchmark}^*$, as when the CoCos have converted:

$$\begin{aligned}\theta_{benchmark}^* &= \theta_{ce}^* = \theta_{pwd}^* \\ n_{benchmark} &= n_{ce} = n_{pwd}\end{aligned}$$

However, in this benchmark case CoCos do not convert so CoCo holders still hold a claim $\bar{e} - \bar{n}$ on the assets that is senior to equity. We have not discussed CoCo pricing, but assume they have a return r_{coco} per unit of investment, then total liability to CoCo holders would be $r_{coco} (\bar{e} - \bar{n})$. Since equity holders are junior to holders of unconverted CoCo's equity holders would each receive

$$\left\{ R_L \left[(1 - n_{benchmark} r_1) - \frac{(\bar{n} - n_{benchmark}) (1 - n_{benchmark} r_1) (1 - \lambda r_1)}{(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1) (1 - \lambda)} \right] - r_{coco} (\bar{e} - \bar{n}) \right\} \times \frac{1}{(1 - \bar{e})} \quad (15)$$

6.2 Convert-to-equity CoCo's (CE)

Suppose returns were R_L and CoCo's converted to equity. Suppose also that the conversion rate of CoCo's is some $\psi > 0$. This allows for the general case where the conversion to equity is not 1 : 1.

Each equity holder now obtains

$$R_L \left[(1 - n_{ce} r_1) - \frac{(\bar{n} - n_{ce}) (1 - n_{ce} r_1) (1 - \lambda r_1)}{(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1) (1 - \lambda)} \right] \left[\frac{1}{(1 - \bar{e}) + \psi (\bar{e} - \bar{n})} \right] \quad (16)$$

We can see that the original equity holders may or may not be wiped out depending on the conversion rate ψ . As $\psi \rightarrow 0$, the original equity holders are not wiped out at all, on the contrary, the CoCo holders are wiped out and the original become the sole residual claimants and obtain all the benefits of the bank remaining afloat. But at the other extreme ($\psi \rightarrow \infty$) old equity holders are completely diluted and the CoCo-holders-turned-equityholders become de facto the sole residual

claimants.

To determine whether a wealth transfer occurs when CoCos convert, we need to compare Eqns. 15 and 16. Since $\theta_{benchmark}^* = \theta_{ce}^*$ and $n_{benchmark} = n_{ce} = n'$ and let $A = (1 - n'r_1) - \frac{(\bar{n}-n')(1-n'r_1)(1-\lambda r_1)}{(1-(\lambda\bar{n}+\frac{e'}{r_1})r_1)(1-\lambda)}$. There will be a wealth transfer from CoCo holders to the original equity holders if the original equity owners' equity position after conversion is better than it is in the benchmark case:

$$\begin{aligned}
R_L A \left[\frac{1}{(1-\bar{e}) + \psi(\bar{e}-\bar{n})} \right] &> \{R_L A - r_{coco}(\bar{e}-\bar{n})\} \times \frac{1}{(1-\bar{e})} & (17) \\
\frac{R_L A}{R_L A - r_{coco}(\bar{e}-\bar{n})} &> \frac{(1-\bar{e}) + \psi(\bar{e}-\bar{n})}{(1-\bar{e})} \\
\frac{R_L A}{R_L A - r_{coco}(\bar{e}-\bar{n})} &> 1 + \frac{\psi(\bar{e}-\bar{n})}{1-\bar{e}} \\
\psi &< \left(\frac{R_L A}{R_L A - r_{coco}(\bar{e}-\bar{n})} - 1 \right) \left(\frac{1-\bar{e}}{\bar{e}-\bar{n}} \right) \\
&= \left(\frac{r_{coco}(\bar{e}-\bar{n})}{R_L A - r_{coco}(\bar{e}-\bar{n})} \right) \left(\frac{1-\bar{e}}{\bar{e}-\bar{n}} \right) \\
&= \frac{r_{coco}(1-\bar{e})}{R_L A - r_{coco}(\bar{e}-\bar{n})}
\end{aligned}$$

Eqn. 17 allows us to define a conversion rate that leaves equity holders in the same position they are in the benchmark case by defining ψ^e as the conversion rate for which Eqn. 17 holds with equality:

$$\psi^e = \left(\frac{r_{coco}(1-\bar{e})}{R_L A - r_{coco}(\bar{e}-\bar{n})} \right) \quad (18)$$

If the conversion rate equals ψ^e , losses due to the asset deterioration are allocated over equity holders and CoCo holders exactly in line with existing seniority. If $\psi > \psi^e$, a wealth transfer takes place from equity holders to CoCo holders; if $\psi < \psi^e$, wealth transfers in fact go the other way, then the original equity holders actually profit from conversion (given that the bad signal has gone out), so the wealth transfer takes place from CoCo holders to the original equity holder. ψ^e depends on both the ratio of unconverted CoCo obligations to the total remaining asset base before CoCos are converted, and the ratio of equity holders to CoCo holders in an obvious way.

Clearly, the direction of the wealth transfer has a major impact on the equityholders' *ex ante* incentives to issue more shares to forestall conversion or on the contrary try to force conversion, for example by organizing well-publicized short selling pressure, depending on whether they are over- or underdiluted compared to the benchmark case. In particular, if $\psi > \psi^e$, the CE CoCo is sufficiently dilutive for old equity holders to make it profitable for them to issue new shares and raise more capital that way to forestall conversion¹³. We can summarize our results in the following proposition:

Proposition 10. *CoCo's that convert to equity potentially effect a wealth transfer from equity holders to the CoCo holders or vice versa depending on the conversion rate. When*

- $\psi < \psi^e = \frac{1-\bar{\epsilon}}{R_L A - (\bar{\epsilon} - \bar{n})}$, *there is insufficient dilution and consequently conversion implies a wealth transfer from CoCo holders to original shareholders.*
- $\psi = \psi^e$, *conversion allocates losses strictly in line with seniority.*
- $\psi > \psi^e$, *CoCos are excessively dilutive, i.e. a wealth transfer from original equity holders to CoCo holders takes place. As $\psi \rightarrow \infty$, CoCo holders wipe out the original shareholders.*

10has major implications for the *ex ante* incentives shareholders and CoCo holders face to forestall or encourage CoCo conversion. The basic idea behind CoCo's is to induce shareholders to issue more equity in times of distress, but the proposition clearly indicates that those incentives may well be absent or even reversed unless CoCo 's are properly designed. Calomiris and Herring (2013) for example stress that CoCos should be properly designed in such a way that their design provides an incentive to shareholders to issue additional equity early enough to forestall conversion. They call for CoCo's to be sufficiently dilutive and postulate that, if designed in that manner, CoCo's will reduce risk shifting pressure by equity holders. They argue that a CoCo, to be sufficiently dilutive, should at least preserve the principal value of the CoCo, and proceed to propose a premium on top of preservation of principal equal to 5% of shareholder value shared out over the entire CoCo base.

¹³If there are costs of equity issuance the trigger level for ψ would change commensurately.

Consider first the preservation of principal (PP) as a guiding principle in setting the conversion ratio. Call the conversion rate that aims to preserve the value of the principal of CoCo's ψ_{PP} . The value of ψ_{PP} follows from equating the principal value of CoCos without a conversion ($\bar{e} - \bar{n}$) and with what the holders of CE CoCos would collectively obtain under conversion, $(R_L A) \left(\frac{1}{(1-\bar{e}) + \psi(\bar{e}-\bar{n})} \right) \psi(\bar{e} - \bar{n})$, which after some algebra yields:

$$\psi_{PP} = \frac{1 - \bar{e}}{R_L A - (\bar{e} - \bar{n})} < \left(\frac{r_{coco}(1 - \bar{e})}{R_L A - r_{coco}(\bar{e} - \bar{n})} \right)$$

$$\rightarrow \psi_{PP} < \psi^e$$

if at least CoCo's earn a positive return, i.e. if $r_{coco} > 1$. So the "Preserving Principal" benchmark value for the conversion rate is very similar to our neutral transfer conversion rate ψ^e but not exactly equal to it. If the net return on CoCo's is zero (i.e. $r_{coco} = 1$), the two trigger rates become the same. That also explains their difference: in the PP case, the original equity holders still manage to appropriate the net return of the CoCo holders and so still obtain some benefits compared to what strict seniority would imply. In our neutrality case where seniority is strictly imposed, CoCo holders receive not only their original principal value but also the statutory returns before equity holders get paid.

Calomiris and Herring (2013), presumably for that reason, propose to pay out a premium over preservation of principal to CoCo holders upon conversion equal to 5% of the equity value at the moment of conversion. We ignore for the moment the problem that for market based triggers (which they prefer) the equity value may in fact reflect that possibility. But even if that is not the case, for example because the trigger is based on book values and the resulting equity values only reflect the residual value conditional on the pre-conversion situation, that rule may still not set incentives in the desired direction. The problem is that the effective return on CoCo's post conversion then depends on the pre-conversion ratio between equity value and CoCo principal. If CoCo's are "large" in respect to equity, 5% of equity value may well not be enough compared to the statutory return on CoCo's r_{coco} . Under the Calomiris Herring proposal, the effective net return

on CoCo's post conversion equals: $r_{CH} - 1 = 0.05 * \frac{(1-\bar{e})}{(\bar{e}-\bar{n})}$. For the Calomiris Herring proposal to actually create *ex ante* incentives to issue more incentives in times of distress, the following needs to hold:

$$\psi_{CH} > \psi^e \Leftrightarrow 0.05 * \frac{(1-\bar{e})}{(\bar{e}-\bar{n})} > r_{coco} - 1 \quad (19)$$

A priori there is no reason to expect this inequality to hold. Note moreover that the likelihood that 19 actually holds goes down as CoCo's become larger. Thus there may well be a conflict between the 5% proposal of Calomiris and Herring and their other proposal, that CoCo's should be "large" in respect to equity. 19 indicates that the larger CoCo's are in relation to equity (i.e. the smaller the ratio $\frac{(1-\bar{e})}{(\bar{e}-\bar{n})}$), the less likely it is that a 5% of total equity value premium to CoCo holders is high enough to provide equity holders with incentives to issue new capital in times of distress. In a sense larger CoCo holdings dilute the 5% equity value that CH want to see transferred to CoCo holders upon conversion. If CoCo holdings become too large (i.e. if $\frac{\bar{e}-\bar{n}}{1-\bar{e}} > \frac{0.05}{r_{coco}-1}$), the *ex ante* incentive for shareholders to issue more equity in times of distress turns perverse, i.e. they get an interest in doing the opposite.

6.3 Principal Writedown CoCo's (PWD)

In Section 4 we described PWD CoCo's in terms of how much of the original CoCo holdings are left after a conversion, as indicated by the remaining fraction φ . CoCo holders under this design have to share what remains of their statutory gross return with the depositors who stayed until $t = 2$, assuming the CoCo claim was not entirely written down (i.e. $\varphi > 0$). Note that the borderline case $\varphi = 1$ corresponds to the benchmark case of section 6.1. Since any value $\varphi < 1$ immediately implies a loss of wealth for the CoCo holder, it is immediately clear that conversion of non-trivial PWD CoCo's (i.e. PWDs with $\varphi < 1$) *always* implies a wealth transfer from CoCo holders to the original equity holder. The existing equity holders do not have to share residual income, if any,

since CoCo holders do not receive any equity under the PWD structure and CoCo holders receive less than they would in the absence of conversion.

Proposition 11. *Conversion of a PWD CoCo always results in a wealth transfer from the CoCo holders to the original equity holders.*

This has major implications for the *ex ante* incentives facing the original equity holders. Contrary to the case of sufficiently dilutive CE CoCo's, under PWD CoCo's the original equity holders always face the perverse incentive that conversion is actually in their interest. This may lead the original equity holders to attempt to extract more dividends or exercise pressure on management to take on more risk in times of distress instead of facing incentives to supply additional capital as in the case of sufficiently dilutive CE CoCo's. The BIS accepts PWD CoCo's even as (Additional) T1 capital (Basel Committee on Banking Supervision (2011)), but that decision should in the light of these incentive problems arguably be reconsidered.

To see whether the original shareholders are better off under CE or PWD for arbitrary values of ψ and φ , we need to compare what each equity holder would get under each case. After conversion of a PWD CoCo, each equity holder now obtains

$$\left\{ R_L \left[\left(1 - n_{pwd}r_1\right) - \frac{\left(\bar{n} - n_{pwd}\right) \left(1 - n_{pwd}r_1\right) \left(1 - \lambda r_1\right)}{\left(1 - \left(\lambda\bar{n} + \frac{e'}{r_1}\right)r_1\right) \left(1 - \lambda\right)} \right] - \varphi \left(\bar{e} - \bar{n}\right) \right\} \left[\frac{1}{\left(1 - \bar{e}\right)} \right] \quad (20)$$

Comparison with an insufficiently dilutive CE CoCo is made easy because we have established that $n_{ce} = n_{pwd}$ by virtue of $\theta_{ce}^* = \theta_{pwd}^*$. Thus equity holders are better off under PWD relative to CE in the sense of them preferring a PWD with retention rate φ over a CE CoCo with dilution parameter ψ if Eqn. 20 is greater than Eqn. 16. Let $n' = n_{ce} = n_{pwd}$ and let $A =$

$R_L \left[(1 - n' r_1) - \frac{(\bar{n} - n')(1 - n'_c r_1)(1 - \lambda r_1)}{(1 - (\lambda \bar{n} + \frac{e'}{r_1}) r_1)(1 - \lambda)} \right]$, this inequality requires:

$$\begin{aligned} \{A - \varphi(\bar{e} - \bar{n})\} \left[\frac{1}{(1 - \bar{e})} \right] &> A \left[\frac{1}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \right] \\ 1 - \frac{\varphi(\bar{e} - \bar{n})}{A} &> \frac{1 - \bar{e}}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \\ \left[1 - \frac{1 - \bar{e}}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \right] \frac{A}{\bar{e} - \bar{n}} &> \varphi \end{aligned}$$

We know that $0 < 1 - \frac{1 - \bar{e}}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} < 1$ and that $\frac{A}{\bar{e} - \bar{n}} > 1$ (if we assume that there is equity left over to distribute). Thus we can always find such a φ for any given value of ψ since $0 < \varphi < 1$ such that the inequality holds, because φ can become very small if necessary. Thus as long as $\varphi < \left[1 - \frac{1 - \bar{e}}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \right] \frac{A}{\bar{e} - \bar{n}}$, equity holders prefer a PWD CoCo with retention rate φ over a CE CoCo with dilution parameter ψ . This is very intuitive because the less of the CoCo's is retained, the lower total liabilities become, and the more is left for the original shareholders. This also means that if the CE CoCo is very dilutive, almost any PWD CoCo would be better for the original equity holders than the CE CoCo.

More generally we can define a retention rate φ^e at which an equity holder would be just indifferent between a PWD CoCo with that retention rate and a CE CoCo with dilution parameter ψ :

$$\varphi^e = \left[1 - \frac{1 - \bar{e}}{(1 - \bar{e}) + \psi(\bar{e} - \bar{n})} \right] \frac{A}{\bar{e} - \bar{n}} \quad (21)$$

with $\frac{\partial \varphi^e}{\partial \psi} = \left[\frac{(1 - \bar{e}) \times (\bar{e} - \bar{n})}{((1 - \bar{e}) + \psi(\bar{e} - \bar{n}))^2} \right] \frac{A}{\bar{e} - \bar{n}} > 0$

In words, a more dilutive CE CoCo will lead equity holders to accept a PWD CoCo with higher retention rate as an alternative.

7 Conclusion

We have written this paper in an effort to explore the effect of (converting) CoCo's on systemic risk. We have done this by adding CoCo holders to the agent types of an otherwise standard Diamond and Dybvig (1983) setup recast in a global games framework as in Goldstein and Pauzner (2005). Using this framework, we were able to show the impact of CoCo conversion on depositors, as well as on regulators and equity holders. First we have shown that when an unanticipated decline in asset returns leads to a CoCo conversion, that has the immediate effect of raising the probability of a bank run. This is true regardless of the type of CoCo's that are converted because they all send the same signal (lowering of returns) which affects depositor incentives in the same manner. Actually we show there is one exception where the CoCo design does matter for the impact on fragility: CoCo's which provide a cash payment to CoCoholders before writing them off (like the RABO CoCo does) are actually worse than straight Principal Write Down (PWD) CoCo's or Equity Conversion (CE) CoCo's in terms of raising the likelihood of a run. This is so because by paying out cash in a distress situation they reduce the amount that may be distributed to the remaining creditors of the bank after conversion occurs.

Therefore one of the main consequences of our analysis is that a regulator faces conflicting incentives when finding out about lower asset returns than expected ($R_L < R$). On the one hand, conversion increases the probability of a run because of the negative signal on asset returns that conversion conveys. But on the other hand, conversion also ensures that if runs occur, there is a higher probability that there will be a surviving equity base. Regulators thus are forced to choose between keeping fragility low at the expense of making the consequences of a run if it does occur worse, or increasing the likelihood of a run but leaving the bank better equipped to deal with one if one occurs.

We then extend the analysis to a multibank framework to analyse the impact of CoCo conversion on systemic risk. When different banks hold assets whose returns are correlated across those different banks, a signal indicating one bank's asset quality deterioration has negative consequences for the other banks to the extent that the other banks' assets are positively correlated to those of

the bank whose CoCo has been forced into conversion: conversion carries an information externality giving rise to contagion across banks. And there is a variety of reasons to expect positive correlation between asset returns of different banks. A very direct link leading to asset correlation establishing a channel of contagion occurs when banks hold each other's CoCos. Given the obvious dangers of contagion such cross holdings give rise to, it is disturbing to see that about 50% of all CoCos issued so far is in fact held by banks (Avdjiev et al. (2013)). Other mechanisms leading to asset correlation maybe the predominance of a few large banks in a relatively small country, industry specialisation of several banks into the same industry, or herding behavior, for example to increase the pressure on regulators to bail out banks in distress if that situation arises. We show unambiguously that in an environment of correlated risks, CoCo conversion, even in a single bank, leads to higher systemic risk, defined as the joint probability of failure of banks. We show that as long as bank assets are positively correlated, a CoCo conversion in one bank leads to an increase in the probability of a run in the other bank, regardless of CoCo type. This implies that systemic risk will increase when CoCos convert. So when regulators consider CoCo conversion, microprudential and macroprudential objectives are likely to be in direct conflict.

Finally we examine the impact of CoCo conversion on wealth transfers between CoCo holders and the original equity holders. This is important because these post-conversion wealth transfers have a direct impact on the pre-conversion incentives for shareholders and CoCo holders to either forestall or actually bring about conversion. From a systemic stability point of view, it is obviously desirable that CoCo design should set incentives in favor of issuing more equity in times of distress. But we show that if the CoCo conversion rate is too low in a well defined sense, i.e. below an explicitly derived trigger level, conversion of CE CoCos actually leads to a wealth transfer from the CoCo holders to the equity holders. This has the perverse consequence that equity holders have an interest in forcing conversion, for example by taking on more risk or extracting cash through excessive dividends. Of course if the conversion rate is higher (more dilutive) than the trigger level, wealth transfers due to conversion go the other way, from the original equity holders to the CoCo holders. So if the conversion rate is sufficiently dilutive in a manner we define precisely, shareholders have an incentive to raise additional capital to forestall conversion. Finally

we show that from the *ex ante* incentive point of view, Principal Write Down CoCo's are strictly inferior to CE CoCo's: their conversion always implies a wealth transfer from CoCo holders to the original equity holder, thereby violating seniority with consequent perverse *ex ante* incentives. These results are important as they point at clear possibly destabilising *ex ante* incentives for equity holders under CoCo design characteristics that are very often observed in practice so far. Our results therefore strongly suggest that the BIS rules governing the requirements CoCo design needs to satisfy before CoCo's are allowed as capital should be reconsidered, particularly for Principal Write Down CoCo's.

References

- Acharya, V. V., Yorulmazer, T., January 2007. Too many to fail—an analysis of time-inconsistency in bank closure policies. *Journal of Financial Intermediation* 16 (1), 1–31.
URL <http://www.sciencedirect.com/science/article/pii/S1042957306000313>
- Acharya, V. V., Yorulmazer, T., 02 2008. Information contagion and bank herding. *Journal of Money, Credit and Banking* 40 (1), 215–231.
URL <http://dx.doi.org/10.1111/j.1538-4616.2008.00110.x>
- Aharony, J., Swary, I., 1983. Contagion effects of bank failures: Evidence from capital markets. *The Journal of Business* 56 (3), pp. 305–322.
URL <http://www.jstor.org/stable/2352800>
- Aharony, J., Swary, I., 1996. Additional evidence on the information-based contagion effects of bank failures. *Journal of Banking and Finance* 20 (1), 57 – 69.
URL <http://www.sciencedirect.com/science/article/pii/0378426694001162>
- Ahnert, T., Georg, C.-P., October 2012. Information contagion and systemic risk.
- Allen, F., Gale, D., 2000. Financial contagion. *Journal of Political Economy* 108 (1), pp. 1–33.
URL <http://www.jstor.org/stable/10.1086/262109>

Avdjiev, S., Kartasheva, A., Bogdanova, B., September 2013. CoCos: a primer. BIS Quarterly Review.

Basel Committee on Banking Supervision, Aug. 2010. Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. Consultative document 174, Bank for International Settlements.

URL <http://www.bis.org/publ/bcbs174.pdf>

Basel Committee on Banking Supervision, Jun. 2011. Basel iii: A global regulatory framework for more resilient banks and banking systems. Tech. Rep. 189, Bank for International Settlements.

URL <http://www.bis.org/publ/bcbs189.pdf>

Berg, T., Kaserer, C., March 2014. Does contingent capital induce excessive risk-taking and prevent an efficient recapitalization of banks?, Systemic Risk, Basel III, Financial Stability and Regulation 2011.

URL <http://ssrn.com/abstract=1709341>

Brunnermeier, M. K., Eisenbach, T., Sannikov, Y., 2013. Macroeconomics with Financial Frictions: A Survey. Cambridge University Press, New York.

Calomiris, C. W., Herring, R. J., 2013. How to design a contingent convertible debt requirement that helps solve our too-big-to-fail problem. *Journal of Applied Corporate Finance* 25 (2), 39–62.

URL <http://dx.doi.org/10.1111/jacf.12015>

Chari, V. V., Jagannathan, R., July 1988. Banking panics, information, and rational expectations equilibrium. *Journal of Finance* 43 (3), 749–61.

URL <http://www.jstor.org/stable/2328198>

Diamond, D. W., Dybvig, P. H., June 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91 (3), 401–19.

URL <http://www.jstor.org/stable/1837095>

- Farhi, E., Tirole, J., February 2012. Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts. *American Economic Review* 102 (1), 60–93.
- Flannery, M. J., 2005. No pain, no gain: Effecting market discipline via “reverse convertible debentures”. working paper subsequently published in Hal S. Scott (ed.), *Capital Adequacy beyond Basel: Banking, Securities, and Insurance* (Oxford: Oxford University Press, 2005).
- Goldstein, I., Pauzner, A., 2005. Demand-deposit contracts and the probability of bank runs. *The Journal of Finance* 60 (3), 1293–1327.
URL <http://dx.doi.org/10.1111/j.1540-6261.2005.00762.x>
- Iannotta, G., Pennacchi, G., 2012. Bank regulation, credit ratings, and systematic risk.
URL <http://ssrn.com/abstract=2021355>
- Jacklin, C. J., Bhattacharya, S., 1988. Distinguishing panics and information-based bank runs: Welfare and policy implications. *Journal of Political Economy* 96 (3), pp. 568–592.
URL <http://www.jstor.org/stable/1830360>
- Lang, L. H., Stulz, R. M., 1992. Contagion and competitive intra-industry effects of bankruptcy announcements: An empirical analysis. *Journal of Financial Economics* 32 (1), 45 – 60.
- Maes, S., Schoutens, W., 2012. Contingent capital: An in-depth discussion. *Review of Banking, Finance and Monetary Economics* 41 (1/2), 59–79.
- Martynova, N., Perotti, E., Oct. 2012. Convertible bonds and bank risk-taking. Tinbergen Institute Discussion Papers 12-106/IV/DSF41, Tinbergen Institute.
- McDonald, R. L., 2013. Contingent capital with a dual price trigger. *Journal of Financial Stability* 9 (2), 230–241.
- Pennacchi, G., 2010. A structural model of contingent bank capital. Working paper, Federal Reserve Bank of Cleveland.
URL <http://ideas.repec.org/p/fip/fedcwp/1004.html>

Sundaresan, S., Wang, Z., 2014. On the design of contingent capital with a market trigger. *The Journal of Finance*.

URL <http://dx.doi.org/10.1111/jofi.12134>

van den Berg, F., Chan, S., van Wijnbergen, S., 2014. An extensive review of currently-issued co-cos. Unpublished manuscript.

Wagner, W., 2010. Diversification at financial institutions and systemic crises. *Journal of Financial Intermediation* 19 (3), 373 – 386.

URL <http://www.sciencedirect.com/science/article/pii/S1042957309000345>

Wilkens, S., Bethke, N., 2014. Contingent convertible ('coco') bonds: A first empirical assessment of selected pricing models. *Financial Analysts Journal* 70 (2).

8 Annex

Calculation of $\frac{\partial \theta^*}{\partial \delta} > 0$

By the implicit function theorem, we can show $\frac{\partial \theta^*}{\partial \delta} > 0$ if $\partial \hat{f} / \partial \theta^* > 0$, and $\partial \hat{f} / \partial \delta < 0$. We already know that $\partial \hat{f} / \partial \theta^* > 0$. We now only look at whether $\partial \hat{f} / \partial \delta > 0$ (here we drop the subscript *cash*

for ease of exposition)

$$\begin{aligned}
\hat{f}(\theta, r_1 \delta) &= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[p(\theta(\theta^*, n)) u \left(\left[\frac{1-\delta r_1 - nr_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} \right] r_D^L \right) - u(r_1) \right] dn \\
&\quad - \int_{n=\frac{1}{r_1}-\delta}^{\bar{n}} u(r_1) \left(\frac{1}{r_1} - \delta \right) \left(\frac{1}{n} \right) \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) u \left(\frac{1-\delta r_1 - nr_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} r_D^L \right) dn - \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} u(r_1) \\
&\quad - \int_{n=\frac{1}{r_1}-\delta}^{\bar{n}} u(r_1) \left(\frac{1}{r_1} - \delta \right) \left(\frac{1}{n} \right) \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) u \left(\frac{1-\delta r_1 - nr_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} r_D^L \right) dn \\
&\quad - u(r_1) \left[\frac{1}{r_1} - \lambda\bar{n} - \frac{e'}{r_1} \right] - u(r_1) \left(\frac{1}{r_1} - \delta \right) \left[\ln \bar{n} - \ln \left(\frac{1}{r_1} - \delta \right) \right]
\end{aligned}$$

where $r_D^L = \frac{1-\lambda r_1}{1-\lambda} R_L$.

The derivative of \hat{f} with respect to δ is

$$\frac{\partial \hat{f}}{\partial \delta} = \frac{\partial}{\partial \delta} \left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) u \left(\frac{1-\delta r_1 - nr_1}{1 - (\lambda\bar{n} + \frac{e'}{r_1}) r_1} r_D^L \right) dn \right] - u(r_1) \left[1 - \left(\ln \frac{\bar{n}}{\frac{1}{r_1} - \delta} \right) \right]$$

where the last term is negative for as long as $0 < \ln \frac{\bar{n}}{\frac{1}{r_1} - \delta} < 1$

Now consider the first term:

$$\begin{aligned}
& \frac{\partial}{\partial \delta} \left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) u(A) dn \right] \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) \frac{\partial u(A)}{\partial \delta} dn + p\left(\theta\left(\theta^*, \frac{1}{r_1}-\delta\right)\right) u\left(\frac{1-\delta r_1-\left(\frac{1}{r_1}-\delta\right)r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) (-1) \\
&\quad - p\left(\theta\left(\theta^*, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right)r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) (-1) \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) \frac{\partial u(A)}{\partial \delta} dn + p\left(\theta\left(\theta^*, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right)r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right)
\end{aligned}$$

where $A = \frac{1-\delta r_1-nr_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L$. But since $\frac{\partial u(A)}{\partial \delta} = \frac{\partial u(A)}{\partial n}$, we can write $\frac{\partial}{\partial \delta} \left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) u(A) dn \right]$ in terms of $\frac{\partial u(A)}{\partial n}$:

$$\begin{aligned}
& \frac{\partial}{\partial \delta} \left[\int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) u(A) dn \right] \\
&= \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} p(\theta(\theta^*, n)) \frac{\partial u}{\partial n} dn + p\left(\theta\left(\theta^*, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right)r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) \\
&= -p\left(\theta\left(\theta^*, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(r_D^L\right) + 2\varepsilon \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[u\left(\frac{1-\delta r_1-nr_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) \left(\frac{p'(\theta(\theta^*, n))}{\bar{n}(1-\lambda)}\right) \right] dn \\
&\quad + p\left(\theta\left(\theta^*, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) u\left(\frac{1-\left(\lambda\bar{n}+\frac{e'}{r_1}-\delta\right)r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) \\
&= -p\left(\theta\left(\theta^*, \lambda\bar{n}+\frac{e'}{r_1}-\delta\right)\right) \left[u\left(r_D^L\right) - u\left(1-\frac{\delta r_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) \right] \\
&\quad + 2\varepsilon \int_{n=\lambda\bar{n}+\frac{e'}{r_1}-\delta}^{\frac{1}{r_1}-\delta} \left[u\left(\frac{1-\delta r_1-nr_1}{1-\left(\lambda\bar{n}+\frac{e'}{r_1}\right)r_1} r_D^L\right) \left(\frac{p'(\theta(\theta^*, n))}{\bar{n}(1-\lambda)}\right) \right] dn
\end{aligned}$$

where we used integration by parts. The first term is clearly negative. The second term can be

made arbitrarily small by letting $\varepsilon \rightarrow 0$.

Thus the derivative of \hat{f} with respect to δ is completely given by

$$\begin{aligned} \frac{\partial \hat{f}}{\partial \delta} &= -p\left(\theta\left(\theta^*, \lambda \bar{n} + \frac{e'}{r_1} - \delta\right)\right) \left[u\left(r_D^L\right) - u\left(1 - \frac{\delta r_1}{1 - \left(\lambda \bar{n} + \frac{e'}{r_1}\right) r_1} r_D^L\right) \right] \\ &\quad + 2\varepsilon \int_{n=\lambda \bar{n} + \frac{e'}{r_1} - \delta}^{\frac{1}{r_1} - \delta} \left[u\left(\frac{1 - \delta r_1 - n r_1}{1 - \left(\lambda \bar{n} + \frac{e'}{r_1}\right) r_1} r_D^L\right) \left(\frac{p'(\theta(\theta^*, n))}{\bar{n}(1 - \lambda)}\right) \right] dn - u(r_1) \left[1 - \left(\ln \frac{\bar{n}}{\frac{1}{r_1} - \delta}\right) \right] \\ &< 0 \end{aligned}$$